

## COMMUNICATION • • • • • • • • • •

The goal stated previously, Goal III, requires the ability to supply explanations. In order to supply an explanation, it must be created or built and then delivered to interested parties. Consider then, where are explanations built?

An explanation is built in a person's mind. It is delivered via communication to others who can accept it, edit it for themselves, help refine it for others, or reject it. Explanations that belong to society's knowledge-base are refined and accepted by many respected individuals. These aspects of working together on knowledge are addressed in the following section.

Human beings communicate and develop language through a variety of mediums: sounds, gestures, marks on a page, tactile differences (e.g. Braille), and artistic works. To avoid ambiguity and added complexity, let the standard for documenting knowledge be written language. In order to further reduce complexity, we'll set the standard to be modern, American English, in keeping with assumption MAE. Consequently, we recognize that communication takes place orally, and in many languages; but to serve the stated goals, this development clarifies communication principles with respect to written, modern, American, English.

Consider what is involved in instances of written communication that provide explanations:

- A. A thought, call it a *target thought*, is formed in someone's mind, such as:
  - The Civil War was fought to abolish slavery
  - force = mass times acceleration
  - the color orange is made from mixing yellow and red together
- B. The target thought is *associated with visually-perceived symbols* such as the following strings of characters:
  - The American Civil War was fought to abolish the institution of slavery.'
  - 'F = ma'
  - 'Orange is made from yellow and red.'
- C. The character strings are *created using a physical medium*, such as ink on paper, chalk on a blackboard, or colored pixels on a computer screen.
- D. Someone else who is interested or motivated in some way *reads the character string and decodes it* to recreate the associated meaning in his or her mind. If the reader attempts to perceive the meaning linked to the written symbols, but does not construct or identify a thought that matches the target thought, then communication of the target thought has not taken place.
- E. A *decision is made* to agree with (accept), edit (partial acceptance and partial rejection), or disagree with (reject) the perceived meaning.

Are any major elements of the communication process missing from this list? Could communication take place without a target thought? Could the sequence of communication events be different? Consider that a string of characters could be effected on a page just for the fun of it, like doodles or as a hobby to fill the time. However, if a character string is used to communicate, then it represents and designates a meaning.

Below are carefully chosen assumptions and vocabulary to govern communication in the work we do together on acquiring knowledge.



### DELIVERING AN EXPLANATION

A meaning is built in the mind of a teacher or author. This meaning, the target thought, is represented via language symbols. The symbols are conveyed in a physical medium to the recipient. Ideally, the recipient decodes the symbols to form a meaning that matches the target thought.

Publishing is an important task for professional scholars. A scholar conveys his or her ideas through authoring a paper that is reviewed by trusted authorities in the relevant field and then is published.



### CONTEXT

Meanings are constructed in a mental construction site called a context.

Some contexts are short-lived and others are developed extensively over time.

Subjects such as chemistry, color, measurement are typically developed in respective contexts.



### HEADQUARTERS [HQ]

Just as Accounting and Human Resources departments provide corporate-wide functions, the capabilities that are present in HQ can affect and interact with all contexts.



Amber married Todd

### ABILITY TO CONCEPTUALIZE

An example of the ability to conceptualize is when we use a verb meaning, to marry, and create a concept from it: marriage. The concept so formed is present in a different context.

## Construction in a Mind

### HQ AXIOM

*A person's mind is a construction site that has stuff, rules, and capabilities present.*

*Def* Context refers to a construction site (or workspace) contained in the mind of a person.

A system of meanings is constructed in a context. Contexts for constructing meanings are derived from areas of human experience such as: family life, politics, sports, economic markets, physical fitness, etc. For example, perceptions and values regarding concern for the environment provide a context and in this setting, the meaning of “pollution” has been built.

*Def* Concept refers to that which has significance or substance in a person's mind, the stuff of a mind that has a defining-limit.

*Def* Conforce refers to a force present in a context.

*Def* A meaning is *initial* if and only if it is a raw material or simple capability present in a context.

*Def* HQ refers to the part of a person's mind that contains everything involved in the ability to communicate (except for a concept of itself).

Observe that when you think, there is a difference between what you are thinking about—for instance, the ideas you are reading about—and all the concepts and memories you have access to. Based on this observation, differentiate between all the meanings a person remembers and has access to in various contexts, and the part of a person's being that actively participates in immediate thought processes.

*Def* Active attention refers to the part of a person's mind which contains short-term memory and is actively engaged in awareness and thinking.

*Def* Memory grid refers to all the meanings that have been defined and/or constructed in a person's HQ.

Also differentiate between the most general context, HQ and other contexts which are part of HQ.

*Def* Sub-HQ context refers to a workspace contained in HQ.

### Forces in HQ

Observation and experience indicate that we have the abilities to:

- attend sequentially
- record direction
- execute or do mental tasks
- duplicate or copy concepts

I assert that we also have the capabilities: to determine, to openize, to conceptualize and to nest-compact, as defined below. They are important for theoretical work. Consider whether or not these capabilities are present in your mind.

*Def* To determine refers to the capability to decide, fix, and establish restrictions in HQ

*Def* To openize refers to the capability to leave open, allow variance, and provide mental space for multiple possibilities or potentialities within a setting contained in HQ

### Conceptualization

In the sentence: “Doug married Ann in 1971,” “married” is a verb. In the sentence, “Ann and Doug's marriage is as strong as ever,” “marriage” is a concept developed from the meaning of “married.” This is an example of conceptualization. Other examples of the ability to conceptualize include the following:

#### MEANINGS EFFECTED BY APPLYING THE ABILITY TO CONCEPTUALIZE

Meaning	Before	After
verb	to act	action
verb	to add	addition
verb	to relate	relationship
verb	to read	reading
adjective	bright	brightness
adverb	silently	silence

In these cases, the word referring to a non-concept meaning changes to function as a noun.

Consider the following. The sentence “she laughed” communicates the meaning referred to by ‘she laughed’. The sentence “He said, ‘she laughed;’” refers to a meaning in which “she laughed” is conceptualized – since “she laughed” is the object of the verb to say and functions as a noun.

Conceptualization involves perception in a different setting or context from the setting in which a force is active. For example, in the context associated with arithmetic, the ability to add is present. Addition, the concept of to add, is not a concept which is used by capabilities in the context of arithmetic – i.e. addition is not added or multiplied. Addition is a concept used in a more general setting to discuss features and qualities of arithmetic. Similarly, “she laughed” is a concept temporally outside the setting in which the laughing took place.

**Def»** [to conceptualize] refers to the capability in HQ such that it uses: a non concept meaning, NC, in a context C  
effects: a concept of NC outside of C

**Sym»** NC

The difference between a setting in which a force is perceived as a means to effect and a setting in which a force is perceived as a thing provides the means to define unique contexts.

As with all capabilities, the direction from what this capability uses, a non-concept meaning, to what it effects, a concept meaning, is an important limit. A non-concept meaning must be available and appropriate before the ability to conceptualize can be applied to it. Ignoring this aspect of conceptualization allows the formation of self-reference paradoxes or contradictions. [See Appendix B for how this pertains to Gödel's work.]

**To Nest-Compact**

Readers who have followed the development so far have formed a concept of HQ. Conceptualization with respect to non-concept meanings in HQ reveals yet another capability our minds have.

When we conceptualize an HQ force, we push the boundaries of what was HQ, so that there is space now for the new concept outside the boundaries of the old HQ. The new HQ then contains the old HQ plus an environment and concept that is outside of the old HQ. The new arrangement

forms the new HQ. This ability will be called the ability *to nest-compact*.

**Def»** [to nest-compact] refers to the capability in HQ such that it uses: a context, CX  
effects: effects a new context N-CX, such that CX is part, not all, of N-CX

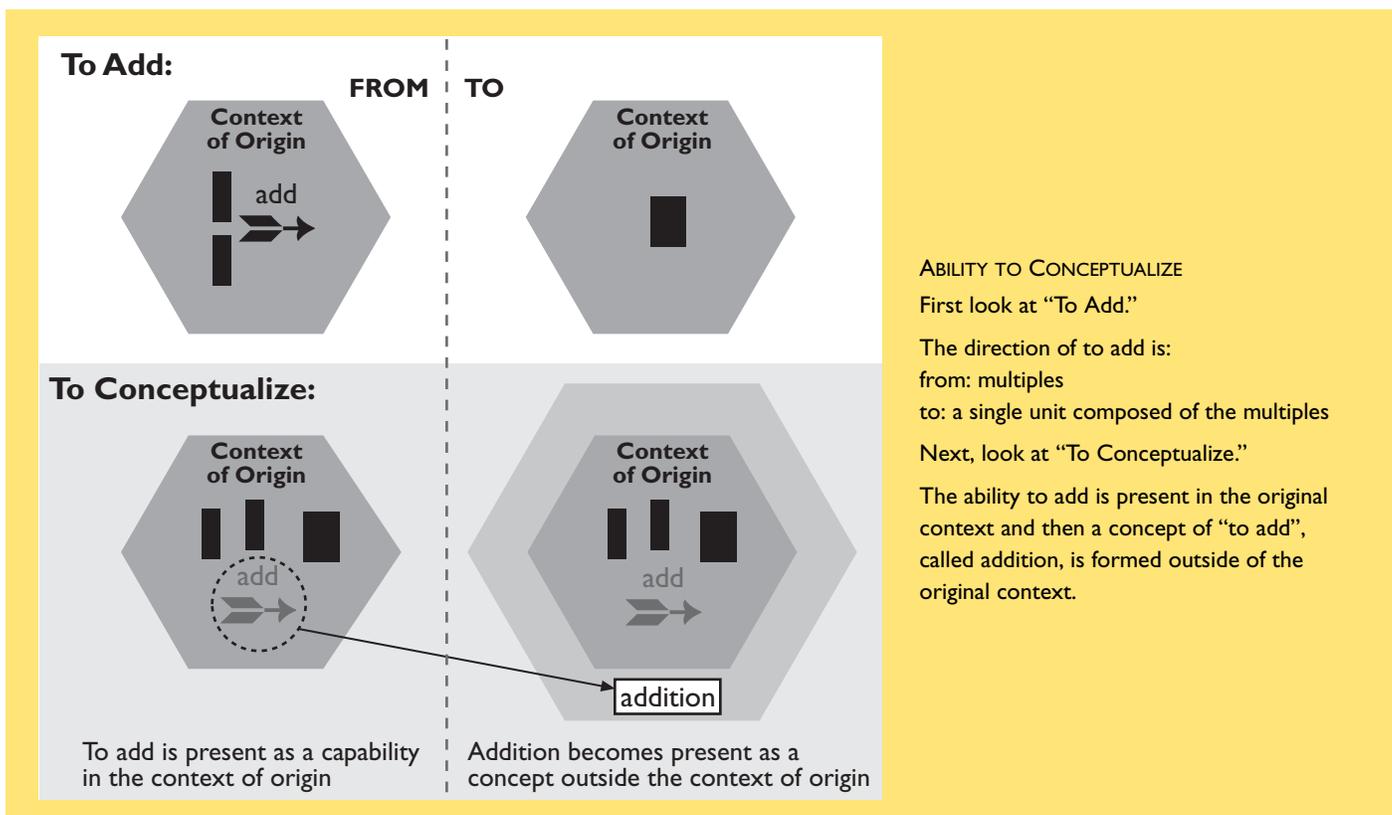
**Sym»** CX

Both the ability to conceptualize and the ability to nest-compact are involved with a scholar's ability to go from the concrete to the abstract. Anecdotal evidence suggests that each scholar has a finite limit for the sequence of iterations that he or she can sustain.

**Capabilities from Prepositions and Conjunctions**

Prepositions and conjunctions are capabilities in HQ which use meanings to effect or build other meanings. The following definitions are meant to capture the conventional role that these words have in English. Allow the following conventions:

- A *type of meaning* refers to whether the meaning is: a concept, force, feature, quality, or a sequence of these as represented by a phrase or sentence.



<b>NOT</b>	<b>OF</b>
<b>AND</b>	<b>/</b>
<b>OR</b>	<b>BY</b>
<b>IF...THEN</b>	<b>WHICH</b>
<b>IF AND ONLY IF</b>	

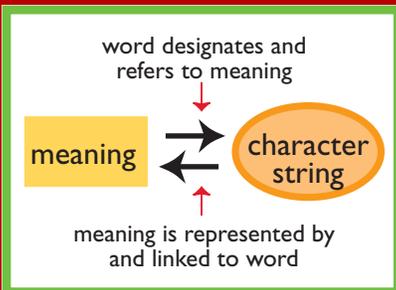
#### PREPOSITIONS AS MEANING TOOLS

Conjunctions and prepositions are tools that provide capabilities in HQ. We'll call these *accepted-HQ-tools*.



#### CHARACTERS

A character is something that can be perceived visually which provides a building block for making words and effecting communication, e.g. punctuation marks..



#### CRITICAL COMMUNICATION LINK

Communication requires that the desired target meaning be represented and designated by a symbol. The symbols of written language are character strings.

- Let different somethings be denoted by Sa and Sb.
- The syntax for applying the following capabilities to the meanings they use is:  
Sa capability Sb

**Def** Not refers to a capability in HQ such that it:

- uses a meaning, Sb
- effects a meaning which satisfies the standard: absence of Sb

**Def** And refers to a capability in HQ such that it:

- uses a group of meanings of the same type
- effects the meaning which satisfies the standards of all the meanings in the group

**Def** Or refers to a capability in HQ such that it:

- uses a group of meanings of the same type
- effects the meaning which satisfies the standard of a meaning contained in the group

**Def** If...then refers to a capability in HQ such that it:

- uses sentence-type meanings with syntax: If Sa, then Sb
- effects: the presence of Sa determines the presence of Sb.

**Def** If and only if [iff] refers to a capa

- uses sentence-type meanings
- effects: If Sa then Sb and if Sb then Sa.

**Def** Of refers to a capability in HQ such that it:

- uses meanings, Sa and Sb
- effects the meaning: Sa as limited or possessed by Sb

**Def** / refers to a capability in HQ such that it:

- uses meanings of the same type
- effects: a whole composed of Sa joined with Sb

**Def** By refers to a capability in HQ such that it:

- uses concepts
- effects: Sb causes or effects Sa

**Def** Which or such that or that refers to a capability in HQ restricted as follows:

- uses Sa and Sb
- restricts Sa according to the standard of the Sb

#### Symbols of Written Language

Observation and experience reveals that the basic building blocks for constructing the symbols of a written language are: letters of an alphabet, punctuation marks and other elementary symbols.

**Def** *Character* refers to a basic unit of visually-perceived differences or visually-perceived sameness that is a raw material in a system of written symbols.

In the system of written symbols used by English, an example of characters which are units of visually-perceived differences are: 'a', 'A', 'z', 'Z', "'". Characters which are units of visually-perceived sameness are: ',', ' ', '■', '□'.

Characters are arranged sequentially in character strings such that the order is understood to be from left to right and then from top to bottom. Thus, a character string is a compound symbol formed from elementary symbols.

#### Symbols and Meanings Together

According to our experience symbols and meanings are put together in order to communicate.

#### COMMLINK AXIOM

*A character string effects communication if and only if it represents and designates a meaning.*

This axiom establishes an association between the character strings and meanings involved in communication such that a char-str points to (designates) a meaning and a meaning points to (is represented by) a char-str.

In the current context of communication, the following standards are hereby established.

We'll say a meaning is *linked to* a char-str if an only if it is represented and designated by the char-str.

Allow that a character string *refers to* a meaning if and only if it designates and represents the meaning.

Meanings are *context-compatible* iff the active context contains the limits used to define them.

## Language Practices

Language rules include: (i) rules which assign a meaning to character strings and (ii) rules governing how character strings are put together to build meaning.

Let *Symbol City* refer to the workspace in the mind where the character strings that refer to meanings are built.

**Def** *Vocabulary definition* [abbreviated voc-def] refers to a rule which:

- a) assumes access to HQ as a source of meaning
- b) sets a standard
- c) assumes the mind's ability to separate meaning which matches the standard from other meaning
- d) takes a finite character string from Symbol City which does not refer to meaning (either it has not been used before or it is unlinked from meaning)
- e) assigns all meanings which satisfy the standard in (b) to the character string indicated in (d)

Thus, a vocabulary definition, or voc-def, is a rule which separates and defines both a unique meaning and a unique written symbol; and then links them together.

Consider that the restrictions or limits supplied by a standard can be established using meanings such that they create a limit like that of a positive image (★) or a negative image (⊖). A positive image standard supplies the pattern that must be matched by focusing on what it is or how it can be built. For example, in studying colors, there are 3 primary colors (raw materials); the definition of "orange" is a color made of red and yellow. On the other hand, there are ways to take advantage of the inherent meaning present in a context and set the pattern which must be matched by associating known meanings to supply the defining limit. For example, this is the case when you learn what a word means from how it is used in a context, the known meanings designated by the other words provide a kind of negative-image standard.

Let *language practice* [lang-prac] refer to a rule for putting names and punctuation marks together in

sequence to effect communication of a meaning.

A language practice is a standard, language construction technique. Language practices include the rules of grammar and syntax which belong to a particular language.

To avoid any ambiguity regarding whether text is used as a voc-def or not, the following language practice is defined.

*Definition of a language practice»*

Use "**Def**" to signify that the instance of written language indented under it is a voc-def.

It is possible to have competing voc-defs in communication. To avoid any ambiguity regarding which voc-def standard a char-str is associated with, the following language practice is defined.

*Definition of a language practice»*

Unless otherwise specified, the most recently communicated voc-def which assigns the char-str meaning is the rule to be used.

### Names

The following terms and standards are hereby established.

**Def** *Name* refers to a character string that represents and designates a meaning according to the rules of a language.

**Def** *Punctuation mark* refers to a character that effects a communication result without referring to meaning.

Spaces and commas are examples of English punctuation marks. They are used to delimit and separate names.

The unique, string of characters which signifies a name can be used in an instance of written language to:

- a) represent and designate its assigned meaning or b) refer to itself as a unit of language. To avoid any ambiguity, the following language practice is defined.

*Definition of a language practice»*

*Name Self Ref*

A name used in an instance of written language to refer to itself as a name and not its assigned meaning, will be offset by single quote marks.



### WAYS OF PROVIDING STANDARDS

Consider that the space for a puzzle piece on the right defines what matches and what does not match. It supplies solid stuff that form the boundaries of the piece being defined. There are ways that we can accomplish the same type of negative-space standard in defining meanings in a context.

The way this happens should become better understood later, in an actual theory.

Artists are familiar with the use of positive and negative space to define shapes. Typically, however, theoretical scientists have not been similarly trained to recognize this aspect of definition in the world of ideas.



### NAMES

"A rose by any other name would smell as sweet" —Shakespeare

A name represents and designates a meaning, it does not impact the meaning itself. When we need to refer to the name as a thing, then we offset the name with single quote marks.

· , ; “ ‘ - (

### PUNCTUATION MARKS AS LANGUAGE TOOLS

Punctuation marks are tools that provide language capabilities.

### Definition of a language practice»

#### First Use

A char-str which refers to itself as it becomes a name through a voc-def will be italicized.

Just as individuals are sometimes given nicknames, a meaning can be assigned an alternate name. Allow the following language practice.

### Definition of a language practice»

#### Alternate Names

To assign an alternate name to meaning which matches the standard set forth by a voc-def, the alternate name is offset with brackets, "[ " and " ]", immediately following the primary name.

### Definition of a language practice:

≡ signifies that the character string or symbol which precedes it is assigned to represent and designate any meaning which matches the standard immediately following it.

A voc-def defines a distinct, unique meaning and separates it from all other possibilities. In many cases, multiple, separate meanings match the standard supplied by a voc-def. For example, the meaning represented by 'planet' is meaning which satisfies the standard: "a non luminous celestial body which orbits a star."<sup>2</sup> The separate meanings designated by 'Earth', 'Mercury', 'Venus', 'Mars' match this standard.

**Def»Constant** ≡ name such that only a single meaning matches the name's voc-def standard.

**Def»Variable** ≡ name such that multiple meanings match the name's voc-def standard.

Examples of constants include: 'Earth', 'Sir Isaac Newton', 'one', and '+'. Examples of variables include: 'planet', 'man', 'number', and 'function'.

Observe that all pronouns are a type of variable. For example, many meanings satisfy the standard for 'he' or 'she'.

There are students of Algebra who have experienced difficulty with understanding the meaning a variable represents. Perhaps this is because to these students the meaning of a

variable appears to be ambiguous – multiple meanings competing for representation by the same name.

Perhaps they could be helped by the improved understanding of how the mechanics of language are working in the context of math.

Each of the individual, specific meanings represented and designated by a variable satisfies the standard associated with the meaning, just like each separate piece of gold meets the standard associated with gold.

And consider that if someone refers to a pie, he or she automatically refers to every piece of the pie. The fact that a piece has been individuated does not cause confusion about whether the piece is pie or which of many pieces is pie or whether only the whole pie is pie. Similarly, the meaning of a variable contains each individuated meaning which satisfies the standard set forth by its voc-def.

**Def»The scope of a name** refers to the group of individuated meanings that a name refers to according to the rules of the written language being used.

**Def»Scp(A)** refers to the scope of "A".

**Def»Let** ↗ refer to a capability.

**Def»Dom(↗)** refers to the domain of ↗.

Observe that a variable can represent meanings which have not been constructed, separated or perceived yet. For example, let 'Monty's Family' refer to Monty, his wife their children and grandchildren. If we know who Monty is, then we know what concepts satisfy this definition; and the meaning of 'Monty's Family' is unambiguous even if no grandchildren have been born yet. Similarly, all planets do not have to be discovered in order to communicate the concept referred to by 'planet'.

#### To Equal and To Patternmatch

When a name and another name both refer to the same meaning, their meanings *equal* each other.

**Def»A equals B** if and only if "A" refers to the same meaning as "B."

**Def»=** ≡ to equal

Equality is about different names referring to the same meaning. The

following is about different meanings having the same pattern of parts.

### Definition of a language practice»

#### Part-Whole Naming

Let "W" name a whole meaning and let "α,..., ω" be a list such that each word in the list names a part of W.

"W{α,..., ω}" refers to the structure or pattern of parts present in W.

**Def»W{α,..., ω} patternmatches** U{β, ..., ψ} if and only if substituting the parts from W in U, U{α,..., ω}, result in the same meaning as W and substituting the parts from U in W, W{β, ..., ψ}, result in the same meaning as U:

W{α,..., ω} = U{α,..., ω} and  
W{β, ..., ψ} = U{β, ..., ψ}

**Def»#** ≡ to patternmatch

## Structure of Written Language

Pieces of written language such as books, articles and paragraphs are constructed using sentences. Each sentence is made with respect to a context. The context dynamically in play for the piece of language currently being communicated will be called the *active* context.

In accordance with the MAE Assumption, you know that sentences are built from units called clauses, and a simple clause is the most elementary kind of sentence. Complex and compound sentences are built from simple clauses using prepositions (e.g. "if," "with," and "such that") and conjunctions (e.g. "and," "or"). Every simple clause is built from a subject and a predicate.<sup>3</sup> Every subject includes a noun and every predicate includes a verb so that the most elementary, simple clause is a single noun followed by a single verb, e.g. 'Seth decided.'

Knowledge is documented with declarative sentences. Thus, a common

3 Reference 4, pp. 2-3, 21-24.

A subject uses a noun with or without modifiers to name multiple somethings or a single something referred to by the Harbrace College Handbook as a person, place, thing, idea, animal, quality, or action [Ref. 4, pp G-27, G-34]. The predicate uses a main verb with or without auxiliary verbs and modifiers to assert or ask something about the subject [Ref. 4, p G-30].

2 Modified version of 'planet' definition from Ref. 2.

understanding regarding simple, declarative sentences, suffices for reasoning together about sentence meanings.

### ATTENDANCE AXIOM

*A simple declarative clause refers to a claim that a result or restriction is present in the active context.*

This axiom asserts that:

- a) each piece of communication is made with respect to a context, the context that is in play or active locally when the statement is communicated
- b) the meaning of a simple declarative clause depends on this active context
- c) a verb meaning - a rule or capability - is included in each simple declarative clause, so each of these clauses identifies a restriction or result.

*Def*» Statement  $\equiv$  declarative clause.

*Def*» A statement is *simple* if and only if the statement only contains a single subject and a single predicate

*Def*» A statement is *true* if and only if its meaning is present in the active context

*Def*» A statement is *false* if and only if it is not true.

In the pursuit of knowledge, we need to maintain clarity about defining-boundaries of the active context. Thus, allow the following standard practice.

*Definition of a language practice*»

Explicitly declare the active context before communicating statements to be evaluated as true or false.

If a context is not specified, assertions are made with respect to the least inclusive context which contains the meanings referred to by the constituent names contained in the statements.

### Logic

Observe that the rules people use for accepting meanings are not necessarily those used in logic. For example, a person may accept the thoughts someone else communicates if the thoughts: correspond to his values, make him feel good, further his

personal agenda, correspond to his intuition, remove an obligation to spend time or money, or correspond to thoughts communicated by authoritative sources. With respect to formal academic knowledge, thoughts are supposed to be accepted or rejected based on whether or not they correspond to what is actually present, i.e. whether or not they are true.

Here, we distinguish between a statement being true in a context, i.e a mental construction site, and a statement being true with respect to the world of human experience. This development deals with the first case. Then subsequently scientists can assess correspondence between our mental constructs and the “real” world.

Logic rules (as established in the work of respected scholars to-date) include: i) rules for assigning 'true' to compound sentence meanings, ii) rules of quantification, and iii) rules which govern proof and justification.

#### Assigning “True”

The following terms are called *propositional connectives*.

*Def*»  $\neg \equiv$  not

*Def*»  $\rightarrow$ [implies]  $\equiv$  if ... then

*Def*»  $\leftrightarrow \equiv$  if and only if

To Be

Allow the following definition to provide the sense of meaning for the verb “to be.”

*Def*» Something *is* something else if and only if the presence of something determines the presence of something else.

#### Truth Tables

A truth table provides a logic rule governing a preposition or to be.<sup>4</sup>

In the following tables, let “A” and “B” refer to meanings that are not everything and not nothing with respect to the active context. “T” refers to true and “F” refers to

false.

B		$\neg B$
T	F	T
F	T	F

A	B	A IS B
T	T	T
T	F	F
F	T	F
F	F	T

A	B	A AND B
T	T	T
T	F	F
F	T	F
F	F	F

A	B	A OR B
T	T	T
T	F	T
F	T	T
F	F	F

A	B	A $\rightarrow$ B
T	T	T
T	F	F
F	T	T
F	F	T

A	B	A $\leftrightarrow$ B
T	T	T
T	F	F
F	T	F
F	F	T

#### Tautologies

*Def*» *tautology*  $\equiv$  a statement containing statement variables which is true for any substitution of (statement) constants for the (statement) variables.

Let A, B, and C each refer independently to different statements. Following are some well-known tautologies.

- (T1)  $(A \rightarrow (B \rightarrow A))$
- (T2)  $(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$
- (T3)  $(B \rightarrow A) \rightarrow ((B \rightarrow A) \rightarrow B)$
- (T4)  $(A \text{ or } \neg A)$
- (T5)  $(A \rightarrow B) \leftrightarrow (\neg B \rightarrow \neg A)$
- (T6)  $(A \leftrightarrow \neg \neg A)$
- (T7)  $(A \leftrightarrow B) \leftrightarrow (\neg B \leftrightarrow \neg A)$
- (T8)  $[(A \rightarrow B) \text{ and } (B \rightarrow C)] \rightarrow (A \rightarrow C)$

<sup>4</sup>“To be” is so common to academic literature that it seems warranted to include it. On the other hand, the language of Arabic - an important respected language - does not have this verb. Perhaps, “to be” is so common that it has become elliptical in Arabic (i.e. understood, not needing to be explicit). “ $\leftrightarrow$ ” joins sentence meanings; “is” joins nouns or a noun with adjective.

## Quantification

Following are some terms established for discussion on these topics.

**Def**  $\forall$  [for all]  $\equiv$  a capability of HQ such that:

- Dom: a variable,  $\chi$ , and a statement, A, which contains  $\chi$
- Syn:  $\forall \chi A\{\chi\}$
- Eff: Meaning of  $A\{\chi\}$  using all unique meanings in the scope of  $\chi$  such that:

**Def**  $\exists$  [for some]  $\equiv$  a capability of HQ such that:

- Dom: a variable,  $\chi$ , and a statement, A, which contains  $\chi$
- Syn:  $\exists \chi A\{\chi\}$
- Eff: Meaning of  $A\{\chi\}$  using some meaning in the scope of  $\chi$  such that:  $\exists \chi A\{\chi\} = \neg \forall \chi \neg A\{\chi\}$   
 $\forall \chi A\{\chi\} = A\{\chi\}$

**Def**  $\emptyset$  [for no]  $\equiv$  a capability of HQ such that:

- Dom: a variable,  $\chi$ , and a statement, A, which contains  $\chi$
- Syn:  $\emptyset \chi A\{\chi\}$
- Eff: Meaning of  $A\{\chi\}$  using no meanings in the scope of  $\chi$  such that:  $\emptyset \chi A\{\chi\} = \forall \chi \neg A\{\chi\}$

**Def** *Quantifier* [Q]  $\equiv \forall, \exists, \text{ or } \emptyset$

Following is the logic rule for a simple statement that contains a variable, x.

*Definition of a language practice*

*Single-Variable Quantification*

$\forall x A\{x\}$  is true if and only if  $A\{c\}$  is true for each constant, c, that satisfies the definition of x.

And this rule applies to statements with multiple variables.

*Definition of a language practice*

*Multi-Variable Quantification*

Let A be a statement which contains only variables  $\chi, \psi, \zeta$  and does not contain quantifiers. Quantification of multiple variables contained in a statement is expressed as follows:

$Q\chi Q\psi Q\zeta A = Q\chi (Q\psi (Q\zeta A))$

$\forall \chi, \psi, \zeta A = A$

$\exists (\chi, \psi, \zeta) A = \neg \forall (\chi, \psi, \zeta) \neg A$

$\emptyset (\chi, \psi, \zeta) A = \forall (\chi, \psi, \zeta) \neg A.$

## Proof

The activity of proving that a statement meaning is true is an activity conducted outside the context of a theory itself. A theory is a system of meanings and a proof answers the question of whether

the meaning of the statement being proved is present in the theory's context or not.

**Def** *Proof*  $\equiv$  a finite, ordered sequence of true statements such that each statement has an acceptable reason to justify why it is true.

A statement is proved means that a proof is supplied such that the statement itself is the last, true statement on the list. Scholars rarely supply all the lines of a proof. Like the pages of a coloring book, only an outline of how the proof goes is supplied; leaving it to the reader to fill in the remaining parts.

*Modus Ponens [MP]*

A common reason for justifying why a statement is true is called *modus ponens* [abbreviated MP] and it works as follows.

Observe in the Truth Table for  $A \rightarrow B$ , that there is only one case in which  $A \rightarrow B$  is true and A is true. In this unique case, B is also true. Therefore, if a statement of the form  $A \rightarrow B$  is true and A is true, then accepting that B is true is justified by the Truth Table for  $A \rightarrow B$ .

*Generalization [GEN]*

A common reason for justifying why a statement is true is called *generalization* and it works as follows.

Given a true statement A that has a free variable,  $\chi$ , the statement  $\forall \chi A$  is true by the Closure Principle. Thus, given A is true, accepting  $\forall \chi A$  is true is justified.

*Deduction*

A common reason for justifying why a statement is true is called *deduction* and it works as follows.

Assume A is true. Prove a statement, B. If the meaning of A is present, then the meaning of B is present—by the proof and the definition of 'if...then'. Thus, given a proof of B such that A is a hypothesis of the proof, accepting  $A \rightarrow B$  is true is justified.

*Contradiction*

A common reason for justifying why a statement is true is called *contradiction* and it works as follows.

A statement A has been proven using a given group of assumptions: R,...,M. Next, a proof is supplied using a group of assumptions, L,...,V, such that R,...,M is contained in L,...,V which justifies  $B \rightarrow \neg A$ .

By tautology T1,  $A \rightarrow (B \rightarrow A)$ . Now, with accepted assumptions L,...,V, A has been proven so by modus ponens:  $B \rightarrow A$ .

By tautology T3,  $(B \rightarrow \neg A) \rightarrow ((B \rightarrow A) \rightarrow \neg B)$ . Per above,  $B \rightarrow \neg A$  has been proven with assumptions L,...,V, so by modus ponens:  $(B \rightarrow A) \rightarrow \neg B$ . And per above,  $B \rightarrow A$  has been proven with assumptions L,...,V, so by modus ponens:  $\neg B$ .

Thus, given L,...,V are true and given A is true and  $B \rightarrow \neg A$  is true in the setting that L,...,V are true, then accepting that  $\neg B$  is true in this setting is justified.

*Example or Counter-Example*

Justification by example or counter-example works as follows.

Given  $\phi$  and a statement A which contains  $\phi$ . Now, let  $\chi$  be a variable such that  $\phi \subseteq \text{Scp}(\chi)$  and  $A\{\chi\}$  be the statement obtained from A by substituting  $\chi$  for  $\phi$  in A. If A is true, then:  $\neg \forall \chi \neg A\{\chi\}$  is true since  $A\{\chi\}$  is true for the meaning of ' $\phi$ ' which is represented by  $\chi$ . Thus, given A is true, accepting  $\exists \chi A\{\chi\}$  is true is justified.  $\phi$  is an "example" of a meaning which makes  $A\{\chi\}$  true.

Similarly,  $\forall \chi A\{\chi\}$  is shown to be false if  $A\{\chi\}$  is false for the meaning of ' $\phi$ ', since  $\phi \subseteq \text{Scp}(\chi)$ . Thus, given A is false, accepting  $\neg \forall \chi A\{\chi\}$  [or  $\exists \chi \neg A\{\chi\}$ ] is justified. In this case,  $\phi$  is a "counter-example" which falsifies  $A\{\chi\}$ .

*Induction*

A common reason for justifying why a statement is true is called *induction* and it works as follows.

Consider a variable  $\chi$  such that for some constant concept  $c_0$  and a capability  $f$ ,  $\chi \equiv c_0$  and if something is  $\chi$ , then  $f$  (something) is  $\chi$ . So the multiple meanings associated with  $\chi$  are built from the initial concept  $c_0$ , executing  $f$  to use  $c_0$  and then executing  $f$  on this

result and so on for successive steps of executing  $f$  so that it uses the result of the previous step. This is the protocol for an *inductive definition*.

Let  $A(\chi)$  represent a statement such that all names are constants except for a variable ' $\chi$ ' that is defined using an inductive definition. Let ' $\tau$ ' represent the initial constant concept and let ' $\varphi$ ' represent the capability used in the inductive definition. Let  $\varphi(\dots\varphi(\tau))$  represent an arbitrary meaning in the scope of ' $\chi$ '. Then " $A\{\varphi(\varphi(\dots\varphi(\tau)))\}$ " represents the next successive meaning for  $\chi$ .

Let  $A(\tau)$  and  $A\{\varphi(\dots\varphi(\tau))\} \rightarrow A\{\varphi(\varphi(\dots\varphi(\tau)))\}$  be true by hypothesis.

CONSTANT MEANING $\subseteq$ SCP(A(x))	REASON WHY STATEMENT MEANING IS TRUE
a) $A\{\tau\}$	hypothesis
b) $A\{\varphi(\tau)\}$	$\tau$ is in the scope of ' $\varphi(\dots\varphi(\tau))$ '; $A\{\varphi(\dots\varphi(\tau))\} \rightarrow A\{\varphi(\varphi(\dots\varphi(\tau)))\}$ } is true due to hypothesis; $A\{\tau\} \rightarrow A\{\varphi(\tau)\}$ is true due to the Contained-In Implication Prin- ciple; then modus ponens.
c) $A\{\varphi(\varphi(\tau))\}$	$\varphi(\tau)$ is in the scope of ' $\varphi(\dots\varphi(\tau))$ ', $A\{\varphi(\dots\varphi(\tau))\} \rightarrow A\{\varphi(\varphi(\dots\varphi(\tau)))\}$ } is true due to hypothesis, $A\{\varphi(\tau)\} \rightarrow A\{\varphi(\varphi(\tau))\}$ is true due to the Contained-In Implica- tion Principle; then modus ponens.
*) $A\{\varphi(\varphi(\dots\varphi(\tau)))\}$	$A\{\varphi(\dots\varphi(\tau))\} \rightarrow A\{\varphi(\varphi(\dots\varphi(\tau)))\}$ } is true due to hypothesis. $A\{\varphi(\dots\varphi(\tau))\}$ is true by previous step. Thus, $A\{\varphi(\varphi(\dots\varphi(\tau)))\}$ is true by modus ponens.

Now, any constant meaning represented by  $A\{\chi\}$  can be represented by  $A\{\tau\}$  or  $A\{\varphi(\varphi(\dots\varphi(\tau)))\}$

—by definitions; and the above table provides justification why each of these statements can be accepted as true. Thus, accepting that  $\forall\chi A\{\chi\}$  is true is justified because each constant meaning represented by  $A\{\chi\}$  is true as shown above.

Therefore, given: a)  $A\{\chi\}$  represents a statement such that the meanings in the scope of ' $\chi$ ' are defined using an inductive definition, b) ' $\tau$ ' represents the initial inductive meaning and c) ' $\varphi$ ' represents the capability used in the inductive definition; if the following statements are both true:

$$A\{\tau\}$$

$$A\{\varphi(\dots\varphi(\tau))\} \rightarrow A\{\varphi(\varphi(\dots\varphi(\tau)))\}$$

then accepting  $\forall\chi A\{\chi\}$  is true is justified.

*Acceptable Reasons Why a Statement is True*

*Def*»Axiom  $\equiv$  a declarative clause that is assumed to be true

Axioms are accepted as true on the grounds that a) we have to start somewhere and b) if axioms are not actually rules of a given system of meanings, then a

### PROOF BY INDUCTION

From Wikipedia (accessed March 2011):  
 “Mathematical induction is a method of mathematical proof ... It is done by proving that the first statement in the infinite sequence of statements is true, and then proving that if any one statement in the infinite sequence of statements is true, then so is the next one.

“Mathematical induction should not be misconstrued as a form of inductive reasoning, which is considered non-rigorous in mathematics ... In fact, mathematical induction is a form of rigorous deductive reasoning.”



DOMINO EFFECT [FROM WIKIPEDIA]

“This method [of proof] works by first proving the statement is true for a starting value, and then proving that the process used to go from one value to the next is valid. If these are both proven, then any value can be obtained by performing the process repeatedly. It may be helpful to think of the domino effect; if one is presented with a long row of dominoes standing on end, one can be sure that:

“The first domino will fall

Whenever a domino falls, its next neighbor will also fall,

so it is concluded that all of the dominoes will fall, and that this fact is inevitable.”

$A \rightarrow B$ $A$ $B$	<i>justi cation</i>
	theorem
	axiom
	modus ponens

### AXIOMS

An axiom is an explicit standard (accepted) assumption. An axiom provides the beginning for a chain of deductive reasoning.

discrepancy will show up eventually which will provide the knowledge that the standard assumptions of a given context are not correct or adequate.

Following are reasons that can be used in a proof to justify that a statement is true. Each can be shown to be a valid justification (when used properly). This is our working list.

- **axiom**
- **hypothesis** – the statement is assumed to be true until specifically stated otherwise
- **modus ponens**
- **generalization**
- **deduction**
- **contradiction**
- **example or counter-example**
- **induction**
- **tautology** – i.e. the statement is constructed in the form of a tautology
- **theorem** – i.e. the statement has been previously proved with respect to the same theory
- **substitution** – i.e. the statement is in the form of a theorem or an axiom that has one or more of its variables replaced with a sub-name (i.e. it is justified by the Contained-In Implication Principle, a theorem or axiom and modus ponens)
- **definition** – the restrictions which determine that the meaning is present are established by the voc-defs and lang-pracs of the language being used.

#### Deductions From False Assumptions

Observe that for all statements A, such that the meaning of A is not everything or nothing with respect to the active context, either A or  $\neg A$ , by tautology T6. Now suppose A is proven using assumptions R,...,M,P such that R,...,M are true and P is false. By deduction,  $P \rightarrow \neg A$  is true for (true) assumptions R,...,M.

Since we assume, that P is true, we can *prove* that A is true by modus ponens. The proof of A using a false assumption allows a false conclusion to be perceived as true.

Therefore, uncertainty about hypotheses leads to uncertainty about the statements proven from these hypotheses.

A modern example of reasoning involving an uncertain, unverified hypothesis is provided in footnote <sup>5</sup>.

#### Terms Regarding Proof of Theory Statements

The following voc-defs are in keeping with the vocabulary used by logicians for discussions regarding the provability of a statement.

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<sup>5</sup> Consider the following example of a conclusion which is accepted as fact even though it is based on an uncertain, unverifiable hypothesis.

Observation and verification of the age of a rock sample uses deductive reasoning which depends on the following hypotheses:

- i) the verifiable measurement (i.e. observation) of an amount of radioactive isotope and its related radioactive decay by-products,
- ii) the verified observation that radioactive isotopes decay at a known, steady rate
- iii) the unverifiable guess regarding the initial state of the isotopes (parent and related daughter isotopes) in the rock at a time labeled  $t_0$
- iv) the unverifiable guess that all decay-related isotopes it currently contains have resulted from the process of radioactive decay

Do we know the condition of the rock at  $t_0$ ? Why should we assume that the rock sample began its life with all stable elements except for trace amounts of a single nuclide/isotope? Couldn't the rock have started out with a mix of isotopes, both parent and daughter? ...or all stable elements? Forces in the cosmos produced the parent nuclide, are we positive this was a once-only process that ended at  $t_0$ ? Do we know for sure that they stopped effecting? Do we know enough about the conditions the rock experienced after  $t_0$  to know that forces present in its environment did not have an impact on the parent/daughter isotopes contained in the rock sample? Hypotheses (iii) and (iv) are ad hoc guesses which are: (a) not axioms of Physics or Cosmology, (b) not deduced from axioms and (c) not verifiable. Consequently hypotheses (iv) and (v) are unreliable. And yet, the age of a rock that is concluded from reasoning using hypotheses (i)-(iv), is reported as fact.

**Def**  $\phi A$  is *provable* iff  $\exists$  proof

- all the true statements listed in the proof are true with respect to a context which contains the meanings referred to by the names of A
- the true statement at the end of the list is A

**Def**  $\phi R, \dots, L \mid_{\text{Th}} A \equiv \exists$  proof

- all the true statements listed in the proof are true with respect to the context named Th
- the true statement at the end of the list is A
- the list R,..., L contains any assumptions used in the proof other than Th's axiom rules.

#### Summary Regarding Communication

In summary, the activity of communication is about definition and construction in a person's mind plus mental tasks related to personal logic.

Names are built and used to represent and designate meaning. The names contained in a language can be set forth outside of a person's mind so that they can be perceived and matched with meaning in the mind of someone else.

It's important to recognize the mind's capability to conceptualize, to openize, and to nest-compact. Prepositions and conjunctions provide capabilities for joining units of meaning.

Many respected scholars have produced a trusted system for evaluating proofs and deductive reasoning in a system called Logic. An overview of key definitions and ideas has been provided. What is new here is a claim that a simple declarative statement is true if and only if the meaning of the statement is present in the active context. Another assertion is that the sense/meaning of "to be" is to determine the presence of (in the active context).

Language and Logic are involved in creating declarative statements plus standards for accepting them, setting them aside for more work, or rejecting them.